Extraction of current density distribution from local magnetic measurements

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We describe a method to extract the current density distribution in a superconducting sample from measurements of the time dependence of the magnetic induction profiles across the sample. Application of this method is demonstrated for a Nd$_{1.85}$Ce$_{0.15}$CuO$_{4.5}$ single crystal in the remanent state.

1. INTRODUCTION

Local magnetic measurements techniques, using miniature Hall-sensor arrays or magneto-optic indicators, have recently proven to be useful in the study of high temperature superconductor (HTS) [1-4]. The raw data provided by these techniques is the position and time dependence of the perpendicular component B$_z$(x,t) of the magnetic induction across the sample surface. These data have been processed to yield important characteristics of flux lines static and dynamics. In all the previous studies, a straightforward determination of the current density distribution was impeded because of lack of knowledge of the in-plane component B$_y$ of the induction field. In this paper we report on a new method to extract the current density distribution J(x,t) from the profiles of perpendicular component B$_z$ alone, utilizing the time dependence of these profiles. This method is applied to a Nd$_{1.85}$Ce$_{0.15}$CuO$_{4.5}$ (NCCO) single crystal in the remanent state.

2. METHOD

We consider a rectangular sample of width 2w (−w ≤ x ≤ w), thickness t (−t ≤ z ≤ 0) and length L >> w, and approximate it as an infinitely long strip with current flowing along its length J=(0,Jy,0). For such a strip E=(0,0,0) and B=(B$_y$,0,B$_z$). We first realize that integration of the measured data B$_y$(x,t) yields E$_y$ directly according to the Maxwell's equation:

$$E_y(x,t) = - \frac{1}{c} \int_0^t \frac{\partial B_z(x',t)}{\partial t} dx'. $$  \hspace{1cm} (1)

Knowing E$_y$(x,t), we proceed to derive J(x,t) assuming a power law model

$$J_y(x,t) = AE_y(x,t)^{1/n} \hspace{1cm} [5,6],$$

where the constants A and n are independent of position. These parameters are determined by substituting this expression into a Bio-Savart formula for B$_y$, and finding the best fit to the experimental data. A convenient expression for B$_y$ is obtained by dividing the sample into N strips of constant integrated current density, $J_i = AE_i^{1/n}$ (i=0..N), as shown in the inset of Fig. 1. For each strip, located at 0 ≤ x$_i$ ≤ |x| ≤ x$_{i+1}$ ≤ w, the z component of the magnetic induction can be calculated analytically, and summation over all current strips gives the total B$_z$:

$$B_z(x,z,t) = H + A \sum_{i=0}^{N-1} \frac{E_i^{1/n}}{c} \int_0^t \left[ \ln \left( \frac{(b_+^2 + g^2)(b_-^2 + g^2)}{(a_+^2 + g^2)(a_-^2 + g^2)} \right) \right] d\zeta \hspace{1cm} (2)$$

where: \hspace{0.5cm} a$_{\pm}$ = x ± x$_i$; \hspace{0.5cm} b$_{\pm}$ = x ± x$_{i+1}$; \hspace{0.5cm} g = z - \zeta , and H is the externally applied field.

We demonstrate this technique in a NCCO single crystal (1.2 × 0.35 × 0.02 mm$^3$) at T=8 K in the remanent state. Figure 1 shows magnetic induction profiles as measured at different times by an array of 11 GaAs/AlGa Hall sensors with 10×10 μm$^2$ active area.
Figure 1. Measured magnetic induction profiles at different times (50, 467, 2706 sec). The solid lines are fits based on Eq. 2. Inset: Schematic illustration of constant current 'strips' within the sample.

From these raw \((x,t)\) data we calculate the electric field distribution \(E(x,t)\) using Eq. 1. The results shown in Fig. 2 indicate a maximum in the electric field near the 'neutral line' \(x/W = 0.7\) where \(B_z\) is independent of time. Knowledge of \(E(x,t)\) enables us to use a two parameters fit procedure to calculate the magnetic induction profiles using Eq. 2. The results of this fitting are shown by the solid curves in Fig. 1. This fitting enables determination of the current distribution \(J_y(x,t)\) as shown in Fig. 3. The figure reveals a non-uniform current density profiles which flattens gradually with time. Note that the maximum current density corresponds to the maximum of the electric field, which is obtained at \(x/W = 0.7\). The inset to Fig. 3 shows the time dependence of the fitting parameters \(A\) and \(n\). The growth of \(n\) with time indicates an approach to a Bean critical state.

In conclusion, we utilized the time dependence of the magnetic induction profiles to extract the current density distribution. Application of this method to a NCCO crystal in the remanent state revealed non-uniform current distribution which relaxes with time towards a uniform, Bean-like current profiles.

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