Effect of nonlinearity on wave scattering and localization

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http://phoi.ifmo.ru/metamaterials//
Costas in APS journals

Search “Soukoulis + xxx (full text)"

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Outline of today’s talk

• Back to 1988
• Disorder and nonlinearity
• Bistability of Anderson states
• Photonic crystals & Fano resonances
• Nonlinear metamaterials
• Self-trapped localized beams
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St. Pnevmatikos (1957-1990)

FIG. 8. Total transmission coefficient $T$ for two-impurity scattering ($M_1 = M_2 = 2$) vs impurities distance $d$ for envelope soliton ($K = 0.8$) in a quartic potential chain ($G = B = 1$). The solid line represents the linear-plane-wave scattering. Numerical points denote the envelope soliton scattering for two different incident energies ($\bigcirc; \varepsilon_1 = 0.0869$, $\triangle; \varepsilon_2 = 0.2373$). $T$ saturates around $T_1 T_2$.

Lattice-soliton scattering in nonlinear atomic chains

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1 MARCH 1988
Localization Decay Induced by Strong Nonlinearity in Disordered Systems

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(Received 11 December 1989)

Linear scattering

Nonlinear scattering
FIG. 1. Logarithm of pulse transmission vs propagation distance. The dashed line is for weak nonlinearity, where the nonlinearity length is greater than the Anderson localization length. The dot-dashed line is for strong nonlinearity, where the nonlinearity length is less than the Anderson localization length. The interesting case, illustrated by the solid line, is when the two lengths are comparable (See Ref. [2]). The symbols are data from this experiment.

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Localized states in disordered structures

What nonlinearity does to such a resonance?

|Ψ| \propto \exp\left(-\frac{|x - x_0|}{\ell_{\text{loc}}}\right)

Anderson 1951
Azbel 1983
Bliokh et al. 2004
Bistability of Anderson Localized States in Nonlinear Random Media

Ilya V. Shadrivov, Konstantin Y. Bliokh, Yuri P. Bliokh, Valentin Freilikher, and Yuri S. Kivshar
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Interference between different photon pathways

Bandwidth modulation with small refractive index variation (\(\delta n/n < 10^{-4}\))

How can we use optical resonators to control light?
Fano resonance


Ranked #3 in Top-100 Phys. Rev. articles!

\[
\varepsilon = 2(\omega - \omega_0)/\Gamma
\]

Fano resonances in nanoscale structures

First published in Nuovo Cimento in 1935
Gustav Mie, 1868-1951

Light Scattering by a Finite Obstacle and Fano Resonances

Michael I. Tribelsky, Sergej Flach, Andrey E. Miroshnichenko, Andrey V. Gorbach, and Yuri S. Kivshar

PRL 100, 043903 (2008)

PHYSICAL REVIEW LETTERS

week ending 1 FEBRUARY 2008
Outline of today’s talk

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Microwave metamaterials from Canberra
Tunability of the ENZ channel

Epsilon-near-zero channel

![Diagram of ENZ channel with tuning elements](image)

(a) Measured and (b) simulated results for different applied voltages and tunable elements.

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**Nonlinear control of tunneling through an epsilon-near-zero channel**

David A. Powell, Andrea Alù, Brian Edwards, Ashkan Vakil, Yuri S. Kivshar, and Nader Engheta

Arrays of nonlinear SRRs

\[ i \frac{\partial \Psi_n}{\partial t} - (2\Omega - i\gamma + \alpha |\Psi_n|^2)\Psi_n - \Sigma = \kappa(\Psi_{n+1} + \Psi_{n-1} - 2\Psi_n) \]


*Bistability, modulational instability, switching waves, dissipative solitons, …*
Moving switching waves

\[ T = 500.00 \]

Moving switching wave

- Plot 1: \(|\psi|^2\) versus \(N_x\)
- Plot 2: \(\varphi/\pi\) versus \(N_x\)
Hysteresis of switching wave velocity

Bistability range: Not standard bistability!

$0.0271 \leq \Sigma \leq 0.03135$

Mechanical analogy: static friction > rolling friction

$V > 0$ means widening of the region corresponding to the upper branch

Bistability range: Not standard bistability!

$0.0271 \leq \Sigma \leq 0.03135$
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Spatial optical solitons
Self-trapping in BEC

Th. Anker et al, PRL 94, 020403 (2005)
Novel ‘broad’ gap states

Self-Trapped Nonlinear Matter Waves in Periodic Potentials

Tristram J. Alexander, Elena A. Ostrovskaya, and Yuri S. Kivshar

truncated nonlinear Bloch modes

two types of modes
Nonadiabatic generation

- Nonadiabatic loading into a 1D optical lattice produces broad states

\[ V_0 = 4E_R; \quad N \sim 10^3 \]
\[ \mu < V_0 \]

Experimental observation in optics

Observation of Nonlinear Self-Trapping of Broad Beams in Defocusing Waveguide Arrays

Francis H. Bennet, Tristram J. Alexander, Franz Haslinger, Arnan Mitchell, Dragomir N. Neshev, and Yuri S. Kivshar
• Nonlinearity produces many interesting effects in wave propagation and scattering
• Many effects have been observed and verified experimentally
• We hope Costas will find more time to study nonlinear effects!!