Loss-compensation in metamaterials

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Major challenges in metamaterials

• Bulk, 3D, isotropy, truly subwavelength structure, bandwidth, ...
• Scaling breaks at optical frequencies
• Nanofabrication, molecular structure

• Dissipative loss

and what we can do about it...

• Loss insensitive designs
  non-resonant metamaterials (e.g. for transformation optics), anisotropic metamaterials
  (focusing, superresolution imaging, “hyperlens”, …)

• Geometric tailoring
  high L low C, big area, smooth corners, bulky designs to avoid skindepth degrading, use extended
  modes for resonators, superconductors, …

• Better materials
  higher plasma frequency, avoid/shift interband transitions, …

• Loss compensation by gain
  electrically/optically pumped gain inclusion coupled to metamaterial resonators
Loss compensation: realistic 4-level gain subsystem

Gain “atoms” (4-level) embedded in host medium:

Driven oscillators at every spatial lattice point which couple to the $E$ field propagated by Maxwell’s equations.

Rate equations:

\[
\frac{\partial N_3}{\partial t} = \Gamma_{\text{pump}} N_0 - \frac{N_3}{\tau_{32}}
\]

\[
\frac{\partial N_2}{\partial t} = \frac{N_3}{\tau_{32}} + \frac{1}{\hbar \omega_a} E \cdot \frac{\partial P}{\partial t} - \frac{N_2}{\tau_{21}}
\]

\[
\frac{\partial N_1}{\partial t} = \frac{N_2}{\tau_{21}} - \frac{1}{\hbar \omega_a} E \cdot \frac{\partial P}{\partial t} - \frac{N_1}{\tau_{10}}
\]

\[
\frac{\partial N_0}{\partial t} = \frac{N_1}{\tau_{10}} - \Gamma_{\text{pump}} N_0
\]

Driven oscillators:

\[
\frac{\partial^2 P}{\partial t^2} + \Gamma \frac{\partial P}{\partial t} + \omega_a^2 P = -\sigma_a \Delta N E
\]

$\sigma_a$ is the coupling strength of $P$ to the external $E$ field and $\Delta N = N_2 - N_1$

Maxwell’s equations:

\[
\nabla \times E = -\frac{\partial B}{\partial t} \\
\nabla \times H = \varepsilon \varepsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t}
\]

FDTD implementation: just more auxiliary equations...

(Here pump rate variant)

Additional polarization from gain atoms in Ampere’s Law:

\[
E^{l+1}_z(i, j) = E^l_z(i, j) + \frac{\Delta t}{\varepsilon_0 \Delta x} \left[ H^{l+1/2}_y(i + 1/2, j) - H^{l+1/2}_y(i - 1/2, j) \right] - \frac{\Delta t}{\varepsilon_0 \Delta y} \left[ H^{l+1/2}_x(i, j + 1/2) - H^{l+1/2}_x(i, j - 1/2) \right]
\]

\[
- \frac{\Delta t}{\varepsilon_0} J_{pz}^{l+1/2}(i, j) - \frac{P^{l+1}_z(i, j) - P^l_z(i, j)}{\varepsilon_0}
\]

Auxiliary equation for gain atom oscillators driven by local E field:

\[
P^{l+1}_z(i, j) = \frac{2 - \omega_a^2 \Delta t^2}{1 + \Gamma \Delta t / 2} P^l_z(i, j) - \frac{1 - \Gamma \Delta t / 2}{1 + \Gamma \Delta t / 2} P^{l-1}_z(i, j) + \frac{\sigma \Delta t^2}{1 + \Gamma \Delta t / 2} \left[ N^l_1(i, j) - N^l_2(i, j) \right] E^l_z(i, j)
\]

Auxiliary rate equations for population inversion:

Non-linearity

\[
N^{l+1}_3(i, j) = \frac{\tau_{32} - \Delta t}{\tau_{32} + \Delta t} N^{l-1}_3(i, j) + \frac{2 \tau_{32} \Delta t}{\tau_{32} + \Delta t} \text{Pr}(i, j) N^l_0(i, j)
\]

\[
N^{l+1}_2(i, j) = \frac{2 \tau_{21} - \Delta t}{2 \tau_{21} + \Delta t} N^l_2(i, j) + \frac{\tau_{21}}{\hbar \omega_a (2 \tau_{21} + \Delta t)} \left[ E^{l+1}_z(i, j) + E^l_z(i, j) \right] \left[ P^{l+1}_z(i, j) - P^l_z(i, j) \right] + \frac{\tau_{21} \Delta t}{(2 \tau_{21} + \Delta t) \tau_{32}} \left[ N^{l+1}_3(i, j) + N^l_3(i, j) \right]
\]

\[
N^{l+1}_1(i, j) = \frac{2 \tau_{10} - \Delta t}{2 \tau_{10} + \Delta t} N^l_1(i, j) - \frac{\tau_{10}}{\hbar \omega_a (2 \tau_{10} + \Delta t)} \left[ E^{l+1}_z(i, j) + E^l_z(i, j) \right] \left[ P^{l+1}_z(i, j) - P^l_z(i, j) \right] + \frac{\tau_{10} \Delta t}{(2 \tau_{10} + \Delta t) \tau_{21}} \left[ N^{l+1}_2(i, j) + N^l_2(i, j) \right]
\]

\[
N^{l+1}_0(i, j) = \frac{2 - \text{Pr}(i, j) \Delta t}{2 + \text{Pr}(i, j) \Delta t} N^l_0(i, j) + \frac{\Delta t}{\tau_{10} [2 + \text{Pr}(i, j) \Delta t]} \left[ N^{l+1}_1(i, j) + N^l_1(i, j) \right]
\]
Optical pumping vs. homogeneous pumping rate

**Optical Pumping**

\[
\begin{align*}
\frac{\partial N_3}{\partial t} &= \frac{1}{\hbar \omega_b} \mathbf{E} \cdot \frac{\partial \mathbf{P}_b}{\partial t} - \frac{N_3}{\tau_{32}}, \\
\frac{\partial N_2}{\partial t} &= \frac{N_3}{\tau_{32}} + \frac{1}{\hbar \omega_a} \mathbf{E} \cdot \frac{\partial \mathbf{P}_a}{\partial t} - \frac{N_2}{\tau_{21}}, \\
\frac{\partial N_1}{\partial t} &= \frac{N_2}{\tau_{21}} - \frac{1}{\hbar \omega_a} \mathbf{E} \cdot \frac{\partial \mathbf{P}_a}{\partial t} - \frac{N_1}{\tau_{10}}, \\
\frac{\partial N_0}{\partial t} &= -\frac{1}{\hbar \omega_b} \mathbf{E} \cdot \frac{\partial \mathbf{P}_b}{\partial t} + \frac{N_1}{\tau_{10}}.
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 \mathbf{P}_i(t)}{\partial t^2} + \Gamma_i \frac{\partial \mathbf{P}_i(t)}{\partial t} + \omega_i^2 \mathbf{P}_i(t) &= -\sigma_i \Delta N_i(t) \mathbf{E}(t) \\
i = a, b; \ \Delta N_a &= N_2 - N_1, \ \Delta N_b &= N_3 - N_0
\end{align*}
\]

\[
\mathbf{P} = \sum_{i=a,b} \mathbf{P}_i
\]

**Homogeneous pumping**

\[
\begin{align*}
\frac{\partial N_3}{\partial t} &= \Gamma_{\text{pump}} \frac{N_0}{\tau_{32}} - \frac{N_3}{\tau_{32}}, \\
\frac{\partial N_2}{\partial t} &= \frac{N_3}{\tau_{32}} + \frac{1}{\hbar \omega_a} \mathbf{E} \cdot \frac{\partial \mathbf{P}_a}{\partial t} - \frac{N_2}{\tau_{21}}, \\
\frac{\partial N_1}{\partial t} &= \frac{N_2}{\tau_{21}} - \frac{1}{\hbar \omega_a} \mathbf{E} \cdot \frac{\partial \mathbf{P}_a}{\partial t} - \frac{N_1}{\tau_{10}}, \\
\frac{\partial N_0}{\partial t} &= \frac{N_1}{\tau_{10}} - \Gamma_{\text{pump}} \frac{N_0}{\tau_{10}}.
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 \mathbf{P}_a(t)}{\partial t^2} + \Gamma_i \frac{\partial \mathbf{P}_a(t)}{\partial t} + \omega_a^2 \mathbf{P}_a(t) &= -\sigma_\Delta N(t) \mathbf{E}(t) \\
\Delta N &= N_2 - N_1
\end{align*}
\]
Optical Pumping

Homogeneous pumping

$P_{\text{in}} = 120.6 \text{ W/mm}^2$

$\Gamma_{\text{pump}} = 9.3 \times 10^9 \text{ s}^{-1}$
Simulation setup

Absorbing boundary condition:  
Perfectly matched layer

Reflected pulse

Incident pulse, plane wave

Structure of interest

Transmitted pulse

Absorbing boundary condition:  
Perfectly matched layer

$e^{ikx}$

$d$

$re^{-ikx}$

$n$

$te^{ikx}$

$z$
2D electric resonance toy model

Lossy electric Lorentz permittivity “wires” as 2D model for electric resonators

\[ \varepsilon = 1 + \frac{\omega_p^2}{(\omega_p^2 - 2i\omega\gamma - \omega^2)} \]

\[ f_p = 100\text{THz}, \; \omega_p = 2\pi f_p; \; \gamma = 2\pi f_c, \; f_c = 10\text{THz}; \; a = 80\text{nm}, \; w_L = 40\text{nm}, \; w_g = 30\text{nm} \]

Gain Bandwidth (20 THz) > Lorentz Bandwidth (5 THz)
Below compensation, \( t = 0.75 \)

Loss compensation!

Gain Bandwidth (5 THz) < Lorentz Bandwidth (20 THz)
Over compensation, \( t = 1.5 \)

2D magnetic resonance toy model

SRR embedded with gain, strong coupling

Gain Bandwidth 20 THz

Resonator “undamping” !!

$\Gamma_{\text{pump}} = 1.0 \times 10^{9} \text{ s}^{-1}$

$\Gamma_{\text{pump}} = 1.4 \times 10^{9} \text{ s}^{-1}$

$\Gamma_{\text{pump}} = 1.9 \times 10^{9} \text{ s}^{-1}$

$a = 100 \text{ nm}$

$l = 80 \text{ nm}$

$t = 5 \text{ nm}$

$d = 4 \text{ nm}$

$w = 15 \text{ nm}$

$\alpha = 100 \text{ nm}$

$l = 80 \text{ nm}$

$t = 5 \text{ nm}$

$d = 4 \text{ nm}$

$w = 15 \text{ nm}$
Gain Bandwidth 20 THz
Emission frequency: 100 THz
Dielectric background of gain: $\varepsilon_g = 2$
Dielectric constant of GaAs: $\varepsilon = 11$
SRR is made of silver: Ordal et al., Drude model

Opt. Exp. 19, 12688 (2011)
buried gain layer beneath the SRR

- more easily implemented (quantum well)
- but needs to be “close” to resonant nearfield; only minor local field enhancement
- gain background

effective homogeneous medium parameter retrieval

microscopic resonant ring current - susceptibility of SRR
embedded gain within gap of SRR

- gain material only in gap
  harder to fabricate at nanoscale
- strongest local field enhancement – easily by a factor > 20
- pure resonator response, very little gain background

effective homogeneous medium parameter retrieval

microscopic resonant ring current – susceptibility of SRR
buried gain, normal incidence: electrically coupled magnetic resonance
Loss-compensated fishnet metamaterial

\[ a_x = a_y = 860 \text{nm}, \quad a_z = 200 \text{nm}, \quad w_x = 565 \text{nm}, \]
\[ w_y = 265 \text{nm}, \quad s = 50 \text{nm}, \quad t = 30 \text{nm}, \quad h_g = 20 \text{nm} \]

Gain bandwidth 20THz
Emission wavelength: 2000nm

FOM = \left| \text{Re} n(\omega) / \text{Im} n(\omega) \right|
Loss-compensated Fishnet at visible frequencies

Permeability $\mu$

$FOM = \left| \frac{Re n(\omega)}{Im n(\omega)} \right|$
Summary & Future work

We compensate loss in metamaterials by inclusions of gain modeled realistically by a 4-level gain system within FDTD simulations

• Linearity window, more gain due to local field enhancement, ...
• Weak coupling yields predominantly “background amplification”
• Strong near-field mediated coupling allows to effectively “undamp” metamaterial resonators: sharpening of magnetic resonance
• Demonstrated loss compensation for realistic 3D structures: SRR with gain in gap or gain beneath surface, Fishnet with intercalated gain (candidate for electric pumping)

Future:

• Spontaneous emission background to realistically model photoluminescence and pump-probe experiments
• Chiral metamaterials with gain
• Better subwavelength structures, ...
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