DENSITY OF TRANSMISSION LEVELS NEAR THE ANDERSON TRANSITION

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Soukoulisfest, Crete, June 2011
THE QUESTION

- Consider transport in simple non-interacting disordered mesoscopic system at zero temperature

- **Weak disorder:** Universal conductance fluctuations, Gaussian distribution of conductance, with \( \text{ave}(g) \gg 1, \) \( \text{var}(g) \sim 1 \) (Stone, Lee, Altshuler, 1985)

- **Strong disorder:** Highly asymmetric distribution
  - Numerical: Soukoulis, Slevin, Markos . . .
  - Analytical: Markos, Wölfle, KAM

**Question:** Properties of the conductance distribution near the Anderson transition?
FEATURES IN QUASI-ONE DIMENSION

• The exact variance has a non-trivial non-monotonic dependence on disorder (Mirlin et al, 1994)

• Distribution is log-normal in deeply insulating regime, but strong deviations near metal-insulator crossover.
STRONG DISORDER IN Q1D: THEORY

KAM & Woelfle (1999)

$\Gamma$ : parameter that describes amount of disorder

$\Gamma = \frac{\zeta}{L}$

Larger $\Gamma$ = smaller disorder

$\Gamma \gg 1$: metal

$\Gamma \ll 1$: insulator

Log-normal distribution at very large disorder

What do you expect when disorder is decreased?
SURPRISES IN Q1D

KAM & Woelfle (1999)

\[ \Gamma = 0.1 \]

\[ \Gamma = 0.25 \]

\[ \Gamma = \frac{\zeta}{L} \]

Sharp cut-off at \( g=1! \)
SURPRISES IN Q1D

KAM & Woelfle (1999)

Highly asymmetric, non-Gaussian distribution!
SURPRISES IN Q1D

KAM & Woelfle (1999)

P[ln(g)]

-5 0 5 10 15 20 25

-ln(g)

Γ = 0.7

Γ = 0.4

Γ = 0.25

Γ = 0.1

Γ = ζ/L

“One-sided” log-normal distribution
THE “LANDAUER” FORMULA

Conductance $g$ via transmission probability $T$:

$$g=\left(\frac{e^2}{h}\right)T$$

commonly known as Landauer Formula

Although first obtained by Economou and Soukoulis

PRL 46, 618 (1981); PRL 47, 973 (1981)
CONDUCTANCE FROM TRANSFER MATRICES

KAM & Woelfle (1999)

Microscopic distribution of impurities
\[ \Rightarrow \text{distribution of transmission levels} \]
\[ \Rightarrow \text{distribution of conductances} \]

Evolution of transmission levels with length:

DMPK in Q1D: Dorokhov (1982), Mello, Pereyra and Kumar (1988):

\[
\frac{\partial p(x, L_z/l)}{\partial L_z/l} = \frac{K}{4} \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left[ \frac{\partial}{\partial x_i} + \frac{\partial \Omega}{\partial x_i} \right] p(x, L_z/l)
\]

\[
\Omega = -\beta \sum_{i<j} \ln |\sinh^2 x_j - \sinh^2 x_i| - \sum_{i} |\sinh 2x_i|
\]

\[
P(g) = \int \prod_{a} dx_a p(x, L_z/l) \delta \left( g - \sum_{i} \text{sech}^2 x_i \right)
\]
EVOLUTION OF TRANSFER MATRICES IN 3D


Relax isotropy assumption:

\[
\frac{\partial p(x, t)}{\partial t} = \frac{1}{4} \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left[ K_{ii} \left( \frac{\partial}{\partial x_i} + \frac{\partial \Omega}{\partial x_i} \right) \right] p(x, t),
\]

\[
\Omega = -\sum_{i<j} \gamma_{ij} \ln|\sinh^2 x_j - \sinh^2 x_i| - \sum_{i} \ln|\sinh 2x_i|.
\]

\[
K_{ab} \equiv \langle k_{ab} \rangle_L; \quad k_{ab} \equiv \sum_{\alpha=1}^{N} |v_{a\alpha}|^2 |v_{b\alpha}|^2; \quad \gamma_{ab} \equiv \frac{2K_{ab}}{K_{aa}}.
\]

For isotropic vectors:

\[
K_{ab}^{Q1D} = \frac{1 + \delta_{ab}}{N+1}, \quad \gamma_{ab}^{Q1D} = 1,
\]

GDMPK reduces to DMPK
PARAMETERS OF GDMPK AT STRONG DISORDER

KAM, Markos & Woelfle (2005)

3D with size $L \times L \times L_z$

$$\Gamma \equiv \frac{l}{K_{11}} L_z = \frac{\zeta}{4} L_z = \frac{1}{Kt} \ll 1$$
$$\gamma \equiv \frac{K_{12}}{K_{11}} = \frac{\zeta}{8} L \ll 1$$

where $\zeta \equiv 4l / K_{11}$ is the localization length

Note: $\Gamma / \gamma = 2 L / L_z$ depends only on geometry

One parameter model for a cubic system
DENSITY OF TRANSMISSION LEVELS IN THE INSULATING REGIME
Douglas & KAM (2009)

Saddlepoint density obeys the integral equation:

\[
\int_{x_1}^{\infty} dy V(x,y)n(y) + U(x) + V(x_1,x) + \ln \frac{n(x)}{N-1} - \Lambda = 0
\]

\[
U(x) = \Gamma x^2 - \ln x - \frac{1}{2} \ln \sinh 2x
\]

\[
V(x,y) = -\frac{\gamma}{2} \ln | \sinh^2 x - \sinh^2 y | - \ln [T_\gamma(x+y)T_\gamma|x-y|]
\]

\[
T_\gamma(x) = 1 - \frac{\gamma \sqrt{Kt}}{\sqrt{2}} erfc \left( \frac{\gamma \sqrt{Kt}}{2\sqrt{2}} + \frac{x}{\sqrt{2Kt}} \right)
\]

\[
erfc(x) = \frac{\sqrt{\pi}}{2} e^{x^2} erfc(x)
\]
DENSITY OF TRANSMISSION LEVELS IN THE INSULATING REGIME

Douglas & KAM (2009)

Strong disorder limit $\Gamma \ll 1$, all $x \gg 1$, $T\gamma = 1$:

Exponential gap at the origin: increases with disorder
compare with constant density with no gap in metals

Opening of the gap could be a signature of the Anderson transition
DENSITY NEAR THE CRITICAL REGIME

Douglas and KAM, 2011

Linear at the critical point

Agrees with numerical works of Markos
CONDUCTANCE DISTRIBUTION IN THE INSULATING LIMIT

Douglas and KAM (2009, 2010)

Strong disorder limit $\Gamma \ll 1$: keep only the smallest transmission level:

$$p(\ln g) \propto \exp \left[ -f \left( \frac{1}{2} \ln \frac{4}{g} \right) \right]$$

$$f(x) = U(x) - \frac{\sqrt{\pi}}{8 e \gamma \sqrt{\Gamma}} \text{erfc} \left[ (x - x_{sp}) \sqrt{\Gamma} \right]$$

Analytic expression as a function of single parameter $\Gamma$
FULL DISTRIBUTION AT VERY LARGE DISORDER

Douglas & KAM (2009)

\[ <\ln g> = -39.4 \quad \text{(Red)} \]
\[ <\ln g> = -15.8 \quad \text{(Blue)} \]
\[ <\ln g> = -08.9 \quad \text{(Green)} \]


Only one parameter that fixes the average
PREDICTIONS FOR VARIANCE


Our asymptotic expression:

$$\text{var}(\ln g) \sim \frac{\langle \ln(1/g) \rangle}{\ln \langle \ln(1/g) \rangle}$$

Explains problems fitting with pure power laws
PREDICTIONS FOR THE THIRD CUMULANT
Douglas & KAM (2009)

asymmetry grows with disorder, within a single parameter theory
PREDICTIONS FOR SKEWNESS: THE TAIL BECOMES THE HEAD!

Douglas & KAM (2009)

Asymmetry starts positive, decreases, goes to zero, changes sign!

Transition at $\langle -\ln g \rangle = 1.3$
SUMMARY AND CONCLUSION

• The generalized DMPK is not very reliable if more than one eigenvalue are close to the origin. However, it should be ok in the critical regime.

• The linear part of the density near the origin agrees with numerical results and theoretical arguments of Markos.

• Should be possible to obtain the full conductance distribution from the density, work in progress.