Quantum Walks and Phase Transitions in Quadratic Nonlinear Waveguide Arrays

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joint work with
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Outline

• Waveguide arrays in material with quadratic nonlinearity
• Second-harmonic generation: Phase transition through parametric interactions; theory and experiment
• Spontaneous parametric down-conversion: Combined photon-pair generation and quantum walks in waveguide arrays
Optical waveguide arrays

- Light tunnels between waveguides
- $E_n$ – amplitude at waveguide number $n$
- $C$ – coupling coefficient

$$i \frac{dE_n}{dz} + C [E_{n-1} + E_{n+1}] = 0$$


Parametric nonlinear interactions

- Periodically poled waveguides in Lithium Niobate
- Linear coupling between waveguides
- Nonlinear coupling between FW and SH through quadratic nonlinearity

$$idA_n/dz + c_{FW}(A_{n+1} + A_{n-1}) + A_n^2 B_n = 0$$

$$idB_n/dz + c_{SH}(B_{n+1} + B_{n-1}) - \Delta \beta B_n + A_n^2 = 0$$
Localization in quadratic layered structures

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Parametric localized modes in quadratic nonlinear photonic structures

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We analyze two-color spatially localized nonlinear modes formed by parametrically coupled fundamental and second-harmonic fields excited at quadratic (Or $\chi^{(2)}$) nonlinear interfaces embedded in a linear layered structure—a quadratic nonlinear photonic crystal. For a periodic lattice of nonlinear interfaces, we derive an effective discrete model for the amplitudes of the fundamental and second-harmonic waves at the interfaces (the so-called discrete $\chi^{(1)}$ equations) and find, numerically and analytically, the spatially localized solutions—discrete gap solitons. For a single nonlinear interface in a linear superlattice, we study the properties of two-color localized modes, and describe both similarities to and differences from quadratic solitons in homogeneous media.

Stationary (soliton) solutions

\begin{align*}
A_i (z) &= A_i (0) \exp[\beta z] \\
B_i (z) &= B_i (0) \exp[2\beta z]
\end{align*}

Small propagation constant $\beta$

\begin{align*}
&\rightarrow \text{wide FW driving „force“} \\
&\text{controls SH} \\
&\rightarrow \text{FW and SH are in phase} \\
&\rightarrow \text{no staggered SH}
\end{align*}

Large propagation constant

\begin{align*}
&\rightarrow \text{narrow FW driving} \\
&\rightarrow \text{FW and SH are out of phase} \\
&\rightarrow \text{staggered SH}
\end{align*}

\begin{align*}
\beta A_i + c_{uu} (A_i A_{i+1}) + \gamma A_i^2 B_i &= 0 \\
-2\beta B_i + c_{uu} (B_i B_{i-1}) - \Delta \beta^2 + \gamma B_i^2 &= 0
\end{align*}

First theory: Sukhorukov, Kivshar, Bang, Soukoulis, PRE 63, 016615 (2001)

Experiment: Setzpfandt, Sukhorukov, Neshev, Schiek, Kivshar, Pertsch, PRL 105, 233905 (2010)
Experimental results

- Mismatch $\Delta \beta$ controlled by FW wavelength
- Spectral peak around $\pi$ in spatial Fourier domain — key feature of staggered phase profile

101 waveguides
15 $\mu$m period
71 mm long
1550 nm laser
5.2 ps pulses

Experiment: Setzpfandt, Sukhorukov, Neshev, Schiek, Kivshar, Pertsch, PRL 105, 233905 (2010)

Spontaneous Parametric Down-Conversion

Spontaneous Parametric Down-Conversion (SPDC) is a common source of entangled photon pairs

Energy conservation
$\omega_{\text{PUMP}} = \omega_s + \omega_i$

Momentum conservation
$k_p = k_s + k_i$

Pump
Nonlinear $\chi^{(2)}$ crystal
$s$ (signal) $\omega_s, k_s$
i (idler) $\omega_i, k_i$
Correlations of photon pairs at the output of the waveguide array

Quantum walks of photon pairs

Correlated photon pairs $a_1^\dagger a_1^\dagger|0\rangle$ at an input lead to output with no classical analogy

Bunching & anti-bunching: signal & idler photons primarily propagate in the same or opposite directions


Simultaneous photon-pair generation and quantum walks in waveguide array with quadratic nonlinearity

Quantum statistics governed by:
• quantum walks - coupling to the right and left
• photon correlations in a pair
• probabilities to generate the photon pairs at different spatial locations

– Pump stays in input waveguide
– Degenerate type-I SPDC

Solntsev, Sukhorukov, Neshev, Kivshar (2011) submitted
Photon pair correlations for pump in one waveguide

**Pump in one waveguide**

Phase-mismatch is equal to 0 when \( k_p^+ \pm k_r^+ \approx \pm \pi \) for \( \Delta \beta^0 = 0 \).

In real space **bunching** and **anti-bunching** are achieved.

Phase mismatch can be controlled by pump wavelength or temperature.

With increase of single waveguide phase mismatch real-space output becomes classical.

Solntsev, Sukhorukov, Neshev, Kivshar (2011) submitted

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Photon pair correlations for pump in two waveguides

**Pump in two neighbouring waveguides with phase shift \( P \)**

In real space bunching and anti-bunching switching can be controlled by phase shift \( P \) between pump waves.

In waveguides \( n = (0,1) \)

Solntsev, Sukhorukov, Neshev, Kivshar (2011) submitted
Fundamental effects and future applications of quadratic nonlinear waveguide arrays

• New type of phase transition associated with second-harmonic generation due to an interplay of localization and synchronisation
  Theory: Sukhorukov, Kivshar, Bang, Soukoulis, PRE 63, 016615 (2001); Experiment in LiNbO$_3$: Setzpfandt et al., PRL 105, 233905 (2010)

• Novel integrated device incorporating spontaneous parametric down-conversion and quantum walks
  – photon pair source with strongly nonclassical correlations with simple classical control of photon statistics by shaping the input pump
  Solntsev, Sukhorukov, Neshev, Kivshar (2011) submitted