Casimir Momentum in Complex Media?

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Costas Soukoulis 60 years, June 2011
Problem of Light Diffusion in Strongly Scattering Media

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Recently, in connection with the problem of Anderson localization of light, a new physical mechanism was considered for the decrease of the diffusion coefficient, in which it is supposed that the velocity of energy transport, which appears in the diffusion constant $D = v_E l_w / 3$, may be much smaller than the phase velocity. Using an exact definition of the diffusion coefficient and generalized Ward identity, we, however, show that in the low-density approximation $v_E$ coincides with the phase velocity.

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There is a considerable interest now in the propagation and multiple scattering of light in strongly disordered media with parameters which approach the assumed threshold of Anderson localization of light, where $\lambda / l < 1$ in the dielectric medium the velocity of electromagnetic energy transport $v_E$, defined as

$$D = v_E l_w / 3,$$

(1)
Localization of classical waves in a random medium: A self-consistent theory

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$c(\omega) \rightarrow 1$. The velocity of energy transport $c(\omega)$ and the phase velocity $c_{ph}(\omega)$ are seen to agree well. In fact, whenever the imaginary part of the mass operator $\Sigma$ is small (e.g., in the limit of low density of scatterers), $c$ and $c_{ph}$ coincide. This is seen immediately from (7) for the case of point scatterers, and was shown in Ref. 11 for general momentum-dependent scattering within a low-density approximation. Therefore, one may conclude that the strong reduction of $c(\omega)$ with respect to $c_{ph}$, as measured experimentally by van Albada et al., is purely an effect of resonant scattering: In the vicinity of resonances $\text{Im}\Sigma$ is always large even for small density and may cause the reduction of $c(\omega)$. 
Quantum Vacuum Contribution to the Momentum of Dielectric Media

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Momentum transfer between matter and electromagnetic field is analyzed. The related equations of motion and conservation laws are derived using relativistic formalism. Their correspondence to various, at first sight self-contradicting, experimental data (the so-called Abraham-Minkowski controversy) is demonstrated. A new, Casimir-like, quantum phenomenon is predicted: contribution of vacuum fluctuations to the motion of dielectric liquids in crossed electric and magnetic fields. Velocities of about 50 nm/s can be expected due to the contribution of high frequency vacuum modes. The proposed phenomenon could be used in the future as an investigating tool for zero fluctuations. Other possible applications lie in fields of microfluidics or precise positioning of micro-objects, e.g., cold atoms or molecules.

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Bi-anisotropic Media

\[ D(\omega) = \varepsilon(\omega)E(\omega) + \chi(\omega) \cdot B(\omega) \]
\[ H(\omega) = \chi^*(\omega) \cdot E + \mu(\omega)B(\omega) \]

Fresnel dispersion law

\[
\begin{align*}
\det \left( \varepsilon \frac{\omega^2}{c_0^2} - p^2 + pp - \frac{\omega}{c_0} \chi \cdot (\varepsilon \cdot p) + \frac{\omega}{c_0} (\varepsilon \cdot p) \cdot \chi^* \right) &= 0
\end{align*}
\]

\[ \chi_{ij}(\omega) = i \omega \ g \delta_{ij} \]
Rotatory power

\[ \chi_{ij}(\omega) = (1 - \varepsilon) \varepsilon_{ijl} \frac{V_l}{c_0} \]
Fizeau effect

\[ \chi_{ij}(\omega) = g \left( E_i^0 B_j^0 - B_i^0 E_j^0 \right) \]
Magneto-electric birefringence

\[ E_0 \times B_0 \]

\[ \omega(\mathbf{k}) \]

\[ \mathbf{k} \]

\[ 10^{-2} \]

\[ 10^{-8} \]

\[ 10^{-15} \]
**phenomenological continuum theory**

\[
\partial_i \left( \rho \mathbf{v} + \frac{1}{4\pi c_0} \mathbf{E} \times \mathbf{B} \right) = -\nabla \cdot \mathbf{T}^0
\]

\[
T_{ij}^0 = \frac{1}{8\pi} \left( \mathbf{E}^2 + \mathbf{B}^2 \right) \delta_{ij} - \frac{1}{4\pi} \left( E_i E_j + B_i B_j \right)
\]

\[
\langle 0 \mid \mathbf{E} \times \mathbf{B} \mid 0 \rangle \propto \begin{cases}
\frac{1}{c_0} \int d^3 k \frac{1}{2} \hbar \omega_k \times g(\omega) \mathbf{E}_0 \times \mathbf{B}_0 = \frac{2}{3} \frac{\hbar \omega_c^4}{\pi^3 c_0^4} g \mathbf{E}_0 \times \mathbf{B}_0 \\
\frac{1}{c_0} \int d^3 k \frac{1}{2} \hbar \omega_k \times [1 - \varepsilon(\omega)] \frac{\mathbf{v}}{c_0} = \rho_{casi} \mathbf{v}
\end{cases}
\]

Inertial mass of quantum vacuum?

Photonic momentum in dielectric media?

→ classical « Abraham » contribution already controversial

UV catastrophe of vacuum energy?

Lorentz invariance of quantum vacuum?

Inertia of quantum vacuum?

cut-off in X-ray?
UV catastrophe in sonoluminescence
(> 1934)

Schwinger (1993)

\[ \Delta E(\text{bubble}) = \int d^3r \left\{ \int d^3k \frac{1}{2} \hbar \omega_k (\text{bubble in water}) - \int d^3k \frac{1}{2} \hbar \omega_k (\text{water no bubble}) \right\} \]

\[ \approx \frac{\hbar a^3}{c^3} \left( \frac{\omega_c^4}{1 - \frac{1}{\sqrt{\varepsilon}}} \right) \approx 10 \text{ MeV} \]

Identity of the van der Waals Force and the Casimir Effect and the Irrelevance of These Phenomena to Sonoluminescence

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The UV catastrophe is real

\[\varepsilon(\omega) - 1 = -\frac{\omega_p^2}{\omega^2} \quad \rho_{\text{casi}} = \frac{\hbar}{c_0^3} \int_0^\infty d\omega\omega^3 \frac{\omega_p^2}{\omega^2} = \infty\]

Free electron

\[g_{\text{ME}}(\omega)/n = -\frac{e^4 m_\Delta^2}{\omega_0^2 m_3 M^2} \left[\frac{\omega^2 + \omega_0^2}{(\omega_0^2 - \omega^2)^2} \left(\mathbf{E}_i^0 \mathbf{B}_j^0 - (\mathbf{E}^0 \cdot \mathbf{B}^0) \delta_{ij}\right) + \frac{1}{\omega_0^2 - \omega^2} \left(\mathbf{E}_i^0 \mathbf{B}_j^0 - \frac{1}{4} \mathbf{E}_j^0 \mathbf{B}_i^0 - \frac{1}{4} (\mathbf{E}^0 \cdot \mathbf{B}^0) \delta_{ij}\right)\right].\]

magnetic dipole

Electric quadrupole

\[g_{\text{ME}} = 10^{-17}-10^{-11}\]

\[P_{\text{casi}} = \frac{\hbar}{c_0^3} \int_0^\infty d\omega\omega^3 g(\omega)\mathbf{E}_0 \times \mathbf{B}_0 = \infty\]

Rizzo et al., 2003-2009, Babington & BAvT, 2011
**Casimir momentum, if infinite, is Lorentz invariant**

\[
L(E, B, v) = -\rho c^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{1}{2} \left( E^2 - B^2 \right) + 2\nu (E^2 - B^2)^2 + \frac{\nu}{2} (E \cdot B)^2
\]

**Bi-anisotropic Lorentz-invariant vacuum**

\[
E = E_0 + E(\omega) \\
B = B_0 + B(\omega)
\]

**Fluctuation-Dissipation**

\[
\langle 0 | E_i(r, \omega) E^*_j(r', \omega') | 0 \rangle = -2\hbar \omega^2 \text{Im} G_{ij}(r, r', \omega) \times 2\pi \delta(\omega - \omega')
\]

**Zero energy flow**

\[
\langle 0 | \frac{E^* \times H}{4\pi} | 0 \rangle = 0
\]

**infinite momentum density**

\[
\langle 0 | \frac{E^* \times B}{4\pi} | 0 \rangle = -\frac{4}{3} \nu K E_0 \times B_0
\]

**Lorentz scalar**

\[
K = \lim_{\omega_c \to \infty} \frac{1}{(2\pi)^3} \sqrt{\frac{1}{2}} \hbar \int_0^{\omega_c} d\omega \int_{4\pi} d\Omega \rho_0(\omega, \Omega)
\]
Classical Abraham momentum in crossed EM fields

\[ \rho v - \mathbf{P} \times \mathbf{B} = \text{constant} = 0 \]

\[ m \mathbf{v}(t) = (\varepsilon - 1) \frac{4}{3} \pi a^3 \mathbf{E}_0(t) \times \mathbf{B}_0 \]

\[ m \ddot{\mathbf{r}}_1 = +q\mathbf{E}(t) + q\dot{\mathbf{r}}_1 \times \mathbf{B} + \mathbf{f}(r_{12}) \]

\[ m \ddot{\mathbf{r}}_2 = -q\mathbf{E}(t) - q\dot{\mathbf{r}}_2 \times \mathbf{B} - \mathbf{f}(r_{12}) \]

\[ 2m \ddot{\mathbf{R}} + q\mathbf{x} \times \mathbf{B} = \text{constant} = 0 \]

\[ m \ddot{\mathbf{x}} = 2q\mathbf{E}(t) + 2q\dot{\mathbf{R}} \times \mathbf{B} - m \omega_0^2 \mathbf{x} \approx 0 \]

\[ 2m \ddot{\mathbf{R}} = \frac{q^2}{m} \frac{1}{\omega_0^2} \mathbf{E}_0(t) \times \mathbf{B}_0 \]
Ex: Helium

$\alpha(0) = 0.22 \times 10^{-40} \text{Cm}^2/\text{V} \quad (16.6a_0^3)$

$\rho = 0.17 \text{kg/m}^3 \text{ (room T)}$

$g = 0.017 \times 10^{-22} \text{m/VT}$

$v_{abr} = \frac{\alpha(0)EB}{2m_p} \approx 3 \text{ nm/ sec}$

$v_{Feigel} = \frac{\pi}{4} \frac{h}{\rho \lambda_c^4} gEB \approx 0.02 \text{ nm/ sec}$

$v_{regula} = -0.0158 \frac{\hbar c_0}{a} (\varepsilon - 1)^2 gEB$

$\approx -0.0 \text{ nm/ sec}$

$v_{QED} = v_{abr} \times \frac{4}{3\pi} \alpha \log \frac{m_{at}}{m_e} \approx 0.08 \text{ nm/ sec}$

Classical abraham force

Feigel QED with cut-off 0.1 nm

Regularization of vacuum energy in $a=10 \text{ cm}$ (Milton, 2000)

QED harmonic oscillator (Kawka, 2010)
\[ p = \alpha(0)E \times B \]

Acoustic pressure

\[ P(\omega) = P_0 + \alpha(0) \times E \times B \times \omega \times \cos \omega t \times n \times L \]

Experiment: Geert Rikken

\[ dp/dt = Abraham \text{ force} \]

V = 8 nm/sec ± 0.8
Feigel: 2 nm/sec

E = 450 V/mm;
B = 1 T;
f = 7.6 kHz
Casimir momentum: $1/4$
QED of harmonic oscillator in crossed fields

$$H = \frac{1}{2m_1}(p_1 - eA_0(r_1) - eA(r_1))^2 + \frac{1}{2m_2}(p_2 + eA_0(r_2) + eA(r_2))^2$$

$$+ eE_0 \cdot r_2 + \frac{1}{2} \mu \omega_0^2 r_{21}^2$$

$$+ \sum_i \hbar \omega_i (a_i^* a_i + \frac{1}{2})$$

$$A_0 = \frac{1}{2} B_0 \times r \quad \phi = -E_0 \cdot r$$
Casimir momentum: $2/4$
QED of harmonic oscillator in crossed fields

Conjugate momenta $\neq$ kinetic momentum

Pseudo-momentum is conserved

\[ p_1 = m_1 v_1 + eA_0(r_1) \]
\[ p_2 = m_1 v_2 - eA_0(r_2) \]

\[ \hat{K} = p_1 + p_2 + \frac{1}{2} eB_0 \times r_{21} = P_{kin} + eB_0 \times r \]

\[ [K, H] = 0 \]
Casimir momentum: \( \frac{3}{4} \)

QED of harmonic oscillator in crossed fields

\[
\hat{K} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 + e \mathbf{A}(r_1) - e \mathbf{A}(r_2) + e \mathbf{B}_0 \times r_{21} + \sum_i \hbar k_i \left( a_i^* a_i + \frac{1}{2} \right)
\]

\[
\langle \Psi_0 | K | \Psi_0 \rangle = M \mathbf{v} + \delta M \mathbf{v} - \frac{e^2}{\mu \omega_0^2} \mathbf{E}_0 \times \mathbf{B}_0 + \delta \left( \frac{1}{\mu} \right) \frac{e^2}{\omega_0^2} \mathbf{E}_0 \times \mathbf{B}_0 + \mathbf{K}_1 + \mathbf{K}_2
\]

\[
\delta M = \delta (m_1 + m_2) \quad \delta \mu = \delta \left( \frac{m_1 m_2}{m_1 + m_2} \right) \quad \delta m_i = \frac{4}{3\pi} \alpha \int_0^\infty dp \frac{p}{p^2 / 2m_i + pc}
\]
Casimir momentum: 4/4

QED of harmonic oscillator in crossed fields

\[ K_1 = \alpha(0)E_0 \times B_0 \frac{m_2 - m_1}{m_2 + m_1} \frac{4}{6\pi} \int_0^\infty dp \left[ \frac{p}{p^2 / 2 + p(m_2c / \hbar)} - \frac{p}{p^2 / 2 + p(m_1c / \hbar)} \right] \]

\[ = \alpha(0)E_0 \times B_0 \frac{m_2 - m_1}{m_2 + m_1} \frac{4}{3\pi} \alpha \log \frac{m_1}{m_2} \]

\[ K_2 \propto \alpha(0)E_0 \times B_0 \alpha \sqrt{\frac{\hbar \omega_0}{\mu c^2}} \propto \alpha^2 \]

\[ K_1: 2\% \, \text{QED correction to Abraham force} \]
\[ K_2: 0.01\% \, \text{QED correction} \]

Kawka & Van Tiggelen, EPL 2010
A quantum vacuum force $F = g \frac{dB}{dt}$?

Chiral geometry with electric polarizabilities

$$\alpha(\omega, \sigma) = \frac{4\pi c^2}{\omega_0^2} \frac{\gamma}{\omega^2 - \omega_0^2 + i\sigma V B + i\gamma\omega_0}$$

Faraday Rotation

$$B = H \Rightarrow \langle 0 | \int d\mathbf{r} \mathbf{E} \times \mathbf{B} | 0 \rangle = \langle 0 | \int d\mathbf{r} \mathbf{E} \times \mathbf{H} | 0 \rangle = 0$$
A quantum vacuum force $F = g \frac{dB}{dt}$?

Chiral geometry with magnetic polarizabilities

$$\chi(\omega, \sigma) = \chi(0) \frac{\omega_0^2}{\omega^2 - \omega_0^2 + i\sigma VB + i\gamma \omega}$$

$$\langle 0 | \int d\mathbf{r} \mathbf{E} \times \mathbf{H} | 0 \rangle = 0$$

$$\langle 0 | \int d\mathbf{r} \mathbf{E} \times \mathbf{B} | 0 \rangle = gB_0$$

Faraday Rotation

$g = \left( \frac{4\hbar c}{3e\mu_0} \right) \left( \alpha^M(0)\mu_0 \right)^5 \left( \frac{1.4 \times 10^4}{L^{14}} \right)$

Na Tetraeder $L=10$ nm $\rightarrow g/m = 1$ nm/sec/T
momentum of quantum vacuum to shed new light on the controversial nature of zero-point energy

Corsica, 2006

Congratulations Costas!