Metamaterials with negative refraction and some relativistic effects.

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Main problem - if we have material with negative refraction, it can be described on the base of negative $n$ and negative $k$ values. The consequences from appearance of negative $n$ are presently more or less clear.
Really, many famous formulas of electrodinamics and optics are not valid for negative $n$, because they are written in so called “nonmagnetic approach”, namely for materials with $B=H$. If $B$ differs with $H$, many formulas should be modified – see the next slide.
<table>
<thead>
<tr>
<th>Physical law</th>
<th>Equation for nonmagnetic approach</th>
<th>Correct equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snellius, Doppler, Cherenkov</td>
<td>( \sin \phi / \sin \psi = n_{21} = \sqrt{\varepsilon_2 / \varepsilon_1} )</td>
<td>( \sin \phi / \sin \psi = n_{21} = \sqrt{\varepsilon_2 \mu_2 / \varepsilon_1 \mu_1} )</td>
</tr>
<tr>
<td>if ( \varepsilon, \mu &lt; 0, ) than ( n &lt; 0 )</td>
<td></td>
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<tr>
<td>Fresnel</td>
<td>( r_\perp = \frac{n_1 \cos \phi - n_2 \cos \psi}{n_1 \cos \phi + n_2 \cos \psi} )</td>
<td>( r_\perp = \frac{Z_2 \cos \phi - Z_1 \cos \psi}{Z_2 \cos \phi + Z_1 \cos \psi} )</td>
</tr>
<tr>
<td>Reflection coefficient for normal fall of light on the border between two media</td>
<td>( r = \frac{(n_1 - n_2)}{(n_1 + n_2)} )</td>
<td>( r = \frac{(Z_2 - Z_1)}{(Z_2 + Z_1)} )</td>
</tr>
<tr>
<td>Condition for full matching</td>
<td>( n_1 = n_2 )</td>
<td>( Z_1 = Z_2 )</td>
</tr>
<tr>
<td>Brewster angle</td>
<td>( \tan \phi = n )</td>
<td>( \tan \phi = \frac{\varepsilon_2 \mu_2 - \varepsilon_1 \mu_1}{\sqrt{\varepsilon_1 \varepsilon_2 \mu_2 - \varepsilon_1 \mu_1}} )</td>
</tr>
</tbody>
</table>
The more complicated question is about negative k. Does it mean, that instead of light pressure, like in vacuum, we have in LHM, following relation $P=\hbar k$, light attraction?

- This problem could not be resolved, if we do not know, what is a value of light pressure in more simple case, namely in materials with positive $n>1$ and $k=n\omega/c > k_0=\omega/c$

- Sorry, at present this problem has many approaches, but not a single convincing decision.
As a first step, let us consider the energy, linear momentum and mass transfer from emitter to receiver in vacuum, following Einstein’s paper A. Einstein, Ann. Phys., 20, 627 (1906).

Time $t$ of passing light from emitter to receiver

$$ t = \frac{L}{c} $$

Momentum $p$ of light pulse gives recoil momentum for emitter

$$ p = \frac{\hbar k}{c} = \frac{E}{c} $$

Velocity of emitter

$$ V = \frac{p}{M} $$

Movement of emitter

$$ X = Vt = \frac{pL}{Mc} $$

Mass $\Delta m$, transferred from emitter to receiver, following center of mass conservation law

$$ MX = \Delta mL $$

$$ \Delta m = \frac{P}{c} = \frac{E}{c^2} $$
Return back to text on previous slide, namely to very famous equation $E=mc^2$.

\[ E=mc^2=m*c*c \]

What does it mean two chars “c”? May be “c” means only constants? Or they have some definite physical meaning?

**Answer** – red equations on previous slide.

One “c” is “$c_{gr}$“ - group velocity of light, the second “c” is “$c_{ph}$“ - phase velocity.

So, two “c” in equation $E=mc^2$ have definite, but different physical meanings, namely phase and group velocities.

So, we have now not $E=mc^2$ but $E=mc_{ph}c_{gr}$

This result is very important!
There is a natural question - what happens if the space between the emitter and the receiver does not fill the vacuum, but a substance with the phase and group velocity $V_{ph}$ and $V_{gr}$?

Should not we in this case replace the equation $E = mc^2$ on $E = mV_{ph}V_{gr}$?

And what about sign of $E$ if $V_{ph}$ and $V_{gr}$ has opposite directions?
Displacement $X$ of emitter for the propagation time $t=L/V_{gr}$ of the object to the receiver.

The movement of mass $M$ to left on distance $X$ should be compensated by the movement of mass $m=P/V_{gr}$ to right on distance $L$.

$X=PL/MV_{gr}$
Let us consider the relations between energy $E$ and linear momentum $P$ of object.

Two different relations are possible:

1. Object is a wave packet (photon, pulse of light).
   \[ P = \frac{E}{v_{ph}} \]
   \[ m = \frac{E}{v_{gr}v_{ph}} \]

2. Object is material body (bullet, elementary particle).
   \[ P = \frac{E}{v_{gr}/c^2} \]
   \[ m = \frac{E}{c^2} \]
Light – wave or particle?

So, there are two sorts of objects with different relations between energy $E$ and linear momentum $P$

1. Wave \[ P = \frac{E}{V_{ph}} = \frac{En}{c} \]
2. Particle \[ P = EV_{gr}/c^2 = \frac{E}{cn} \]

We assume absence of frequency and spatial dispersion, so \[ V_{ph} = V_{gr} = \frac{c}{n} \]

WHY TENSORS?
Tensors are the sources for calculations of ponderomotive forces

Both tensors have similar component, except value of linear momentum density $g$

$$f_i = \frac{\partial T_{ik}}{\partial x_k}$$

$$T_{ik} = \begin{bmatrix} \theta_{\alpha\beta} & gc \\ S/c & W \end{bmatrix}$$

$i, k = 1, 2, 3, 4$

$\alpha, \beta = 1, 2, 3$
Energy-momentum tensors

Two possible realisations from years about about 1904

\[ \mathbf{T}_{\kappa} = \begin{bmatrix} \theta_{\alpha\beta} & gc \\ S/c & W \end{bmatrix} \]

**Minkowski**

\[ \theta_{\alpha\beta} = \frac{1}{4\pi} (E_\alpha D_\beta + H_\alpha B_\beta) - \frac{1}{8\pi} \delta_{\alpha\beta} (ED + HB) \]

\[ S = \frac{c}{4\pi} [EH] \]

\[ g = \frac{1}{4\pi c} [BD] \]

\[ W = \frac{1}{8\pi} (ED + HB) \]

**Abrakham**

\[ \theta_{\alpha\beta} = \frac{1}{4\pi} (E_\alpha D_\beta + H_\alpha B_\beta) + E_\beta D_\alpha + H_\beta B_\alpha \]

\[ - \frac{1}{8\pi} \delta_{\alpha\beta} (ED + HB) \]

\[ S = \frac{c}{4\pi} [EH] \]

\[ g = \frac{1}{4\pi c} [EH] \]

\[ W = \frac{1}{8\pi} (ED + HB) \]
CONCLUSIONS

1. Relation between the mass $m$ of a transferred light and its energy $E$ in the general case is $m = \frac{E}{V_{ph} V_{gr}}$. The well-known relation $m = \frac{E}{c^2}$ is a special case.

2. Abraham tensor is not relativistically invariant and cannot be used for calculation of the forces of light pressure. This fact completely resolves the century-old debate about “Minkowsky – Abrakham controversy”.

3. In materials with negative phase velocity does not occur light pressure but presents light attraction. In this case the mass transferred not from the emitter to the receiver, but from receiver to emitter.
The main stages in the development of new scientific ideas

- This could not be, because this can not be ever!
- All this is true, but long been known.
- Are you here with?
About history of negative refraction…

\[(\varepsilon, \mu < 0) \Rightarrow (n < 0) \Rightarrow (\frac{v_{ph}}{v_{gr}}) \Rightarrow \text{Diagonal}\]

- **Veselago, 1966-1968**
- **Sivukhin, 1957**
- **Pafomov, 1959**
- **Maluzhinets, 1951** (Transmission lines)
- **Backward wave tube, ~1950**
- **Shuster, 1904**
- **Mandelstam, 1944** («Photonic crystals»)
- **Lamb, 1904; Pocklington, 1905** («Photonic crystals»)
What is the difference?

- Abrakham tensor is not relativistic invariant, but Minkowski tensor is. So, Abrakham tensor is not a tensor.
- Both tensors are not valid for materials with negative $\varepsilon$ and $\mu$, because of negative sign before energy density $W$. 
Main questions:
1. What is a pressure on the interface between slab and emitter (E) and receiver (R)?
2. What is a mass, transferred from (E) to (R), if wave packet with energy $E$ transferred from (E) to (R) inside the slab of media?
Let us consider a little modified picture from previous slide. Let us include some small vacuum gaps between slab and emitter and receiver. By this choice of geometry we could calculate separately the forces on the interfaces of slab, emitter and receiver. As to forces on the interfaces of emitter and receiver, inserted in vacuum, one could estimate it, following our first slide.
If the wish to save center of mass conservation and equation $E=mc^2$, we need to write $P=hw/nc$, not $P=hnw/c$, and it is a «mechanical approach».

If momentum of light is proportional to $nhw/c$, the center of mass conservation and Einstein relation $E=mc^2$ are not consistent. This statement based on «quantum approach».
Now the have very important contradiction.

If we prefer to have mass, transferred from emitter to receiver as $E/c^2$, we need consider momentum of light $P$ inside media as $P=E/cn_{gr}$.

This value does not depend on direction of vector $K$, and is equal for LHM and RHM.