The Twists and Turns of Chiral Swiss Rolls

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Why Swiss Rolls?

• Frequency range 10 – 300 MHz, i.e. wavelength range 30 – 1 metre
• So $\lambda / a \sim 1000$ can easily be achieved:
  – Homogenization and the effective medium description should be valid.
• They are strongly “magnetic” metamaterials :-
  – Negative bandwidth $(\omega_p - \omega_0) / \omega_0 > 40$
• Measurements can be made at $r < \lambda / 100$, i.e.
  – In the extreme near field
  – $E$ and $H$ are decoupled
  – $\varepsilon$ controls $E$; $\mu$ controls $H$
• MRI application:-
  – Metamaterials can be magnetic at RF but non- magnetic at DC
  – Use for RF flux manipulation: guiding, shielding, etc.
• They can exhibit massive Chirality
  – Negative $n$ with a single component
Chiral Swiss Rolls

- Plain & ordinary
- Anti-clockwise spiral
- Left handed

- Ornate & exotic
- Clockwise spiral
- Right handed

- But of course these are the same (just rotate by 180°)!

- To distinguish them, we must pull out the middle
Chiral Swiss Rolls in practice

- Wind a continuous strip, of width $w$, at an angle $\theta$, to give a pitch $p = 2\pi r \tan \theta$, so that the number of local turns $N = w/p$
- In practice, winding a 5mm tape with 2° offset on a 5mm mandrel gives $N \approx 9$ and a resonance at $\sim 80$ MHz
**Single roll summary**

- We record the resonant frequency from the winding machine

<table>
<thead>
<tr>
<th>Series</th>
<th>Mandrel</th>
<th>Sense</th>
<th>Angle</th>
<th>Frequency / Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5.35 mm</td>
<td>Square</td>
<td>0°</td>
<td>145</td>
</tr>
<tr>
<td>B</td>
<td>5.35 mm</td>
<td>L-R</td>
<td>5°</td>
<td>148</td>
</tr>
<tr>
<td>D</td>
<td>5.35 mm</td>
<td>R-L</td>
<td>5°</td>
<td>142</td>
</tr>
<tr>
<td>E</td>
<td>5.0 mm</td>
<td>R-L</td>
<td>5°</td>
<td>150</td>
</tr>
<tr>
<td>F</td>
<td>5.0 mm</td>
<td>R-L</td>
<td>4°</td>
<td>130</td>
</tr>
<tr>
<td>G</td>
<td>5.0 mm</td>
<td>R-L</td>
<td>3°</td>
<td>110</td>
</tr>
<tr>
<td>M</td>
<td>5.0 mm</td>
<td>R-L</td>
<td>2°</td>
<td>81</td>
</tr>
<tr>
<td>P</td>
<td>5.0 mm</td>
<td>L-R</td>
<td>2°</td>
<td>81</td>
</tr>
<tr>
<td>H</td>
<td>4.0 mm</td>
<td>R-L</td>
<td>5°</td>
<td>155</td>
</tr>
<tr>
<td>K</td>
<td>4.0 mm</td>
<td>L-R</td>
<td>5°</td>
<td>160</td>
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<tr>
<td>J</td>
<td>4.0 mm</td>
<td>R-L</td>
<td>4°</td>
<td>142</td>
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<tr>
<td>L</td>
<td>4.0 mm</td>
<td>R-L</td>
<td>3°</td>
<td>120</td>
</tr>
<tr>
<td>N</td>
<td>3.0 mm</td>
<td>R-L</td>
<td>5°</td>
<td>202</td>
</tr>
<tr>
<td>O</td>
<td>3.0 mm</td>
<td>R-L</td>
<td>4°</td>
<td>172</td>
</tr>
</tbody>
</table>

- Frequency depends on mandrel diameter and winding angle
- All consistent with results & expectations from non-chiral rolls
What will we see?

• The constitutive relation for a general linear medium is

\[
\begin{bmatrix}
D_x \\
B_z
\end{bmatrix}
= \begin{bmatrix}
\xi & E \\
\mu & H
\end{bmatrix}
\begin{bmatrix}
E \\
H
\end{bmatrix}
= EM
\]

• For a chiral medium, \( \varepsilon, \mu, \xi = -i\kappa \) and \( \zeta = i\kappa \) are diagonal

• If we apply a magnetic field \( H \), we should observe both \( B = \mu H \) and \( D = -i\kappa H \) responses, parallel to the applied field, i.e. there should be an electric field response \( E_z \) to a magnetic excitation \( H_z \) and vice versa
Measuring Constitutive Parameters ($\varepsilon$, $\mu$, $\kappa$)

\[
\begin{bmatrix}
D \\
B 
\end{bmatrix} = \begin{bmatrix}
\varepsilon & E \\
\mu & H 
\end{bmatrix} \begin{bmatrix}
E \\
H 
\end{bmatrix} = EM
\]

- Use network analyser measuring $S_{21}$ to determine:

- Permeability $\mu$
  - Apply a uniform $H$ and measure the $B$ induced in a loop.

- Permittivity $\varepsilon$
  - Apply a uniform $E$, and measure $D$ via the impedance of a “conventional” parallel plate capacitor.

- Chirality $\kappa$
  - Apply a uniform $E$ and measure the induced $B$ with a loop.
Single Elements
Single rolls

- No resonance in $\varepsilon$ or $\kappa$ from non-chiral rolls
- Dielectric and chiral response observed from chiral rolls
- Chiral response is inverted for opposite handed rolls

Chirality is bigger than $\mu$ or $\varepsilon$
Further Characterisation: Z-F maps

Non-chiral roll

Chiral roll, 2°
R-L, 5mm mandrel
Modes in M-series (5 mm, 2 deg, R-L)
Mode dispersion in M-series (5 mm, 2 deg, R-L)
Pairs
Pairs of chiral rolls in a magnetic field

- The rolls behave as (large) electric and magnetic dipoles ($\mathbf{P}$ and $\mathbf{M}$)
  
  \[ \mathbf{P} = \mathbf{H} \sqrt{\varepsilon_0 \mu_0} \mathbf{M} \quad \text{and} \quad \mathbf{H} = ( -1 ) \]

- The energy of one big dipole (2 point charges separated by $L$) in the field of another a distance ($d + d_0$) away is

  \[ U = A \left( \frac{1}{\sqrt{L^2 + (d + d_0)^2}} - \frac{1}{(d + d_0)} \right) \]

  \[ A = \frac{P^2}{4\pi\varepsilon_0} \quad \text{for electric dipoles and} \quad A = \frac{\mu_0 M^2}{4\pi} \quad \text{for magnetic dipoles} \]

- Confirm description by plotting peak frequency shift vs. separation $d$, and fit for $A$ and $d_0$

- Find
  
  $A = -13.6 \text{ MHz.mm}$

  $d_0 = 0.54 \text{ mm}$

  $r^2 = 0.999$
Pairs – magnetic responses

- The frequency shifts are due to the coupling between the elements
- The chiral pair has the magnetic and electric dipoles with the same sense
  \[ U_C = U_M + U_E \]
- Peak frequency increased
- For the enantiomeric pair the electric dipoles are reversed
  \[ U_R = U_M - U_E \]
- Peak frequency decreased
- No chiral response
- From the data we find
  \[ U_C = 3.7 \text{ MHz},\ U_R = -17.3 \text{ MHz},\ U_M = -6.8 \text{ MHz},\ U_E = 10.5 \text{ MHz} \]
- Ratio is
  \[ \frac{U_E}{U_M} = \left( \frac{\kappa}{\mu - 1} \right)^2;\ \frac{\kappa}{\mu - 1} = 1.24 \]
- Consistent with single data
Quads
Quads – magnetic responses

- LH and RH Quads show opposite chirality
- No chirality for enantiomeric quad
- Resonant frequency for LH & RH Quads shifted 2.5 MHz higher than for the pairs: all dipoles are parallel
- Frequency for the enantiomeric quad is depressed by a further 12 MHz: electric and magnetic dipoles in opposite directions
- Ratio of magnetic to electric energy is
  \[
  \frac{U_E}{U_M} = \left( \frac{\kappa}{\mu - 1} \right)^2; \quad \frac{\kappa}{\mu - 1} = 1.24
  \]
- Same as for pairs
Cubes
Axial cubes – magnetic responses

- Non-chiral cube appears to have increased resonant frequency over single roll
- But no demagnetisation correction has been applied
- This is $D \sim 1/3$ but is susceptibility dependent for uniaxial anisotropic cube.
- This correction gives resonant frequency approx equal to singles.
- Applying correction to chiral cube gives frequency above single (95.3 MHz cf 82.7 MHz)
- Consistent with expectation from band structure calculation
- Frequency for the enantiomeric cube is further depressed
- Racemic mixture is NOT the same as non-chiral!
Conclusions

• Chiral materials have a constitutive equation of the form
  \[
  \begin{bmatrix}
  D_x \\
  B_x
  \end{bmatrix} = \begin{bmatrix}
  \kappa & E_x \\
  \mu - i & H
  \end{bmatrix} \begin{bmatrix}
  D_y \\
  B_y
  \end{bmatrix}
  \]

• Chiral Swiss Rolls have been made and characterised
• They show strong chiral behaviour which is directly observable: we find a magnetic response to an electric field
• Pairs, quads, and cubes show increasingly complex behaviour
• Enantiomeric or racemic mixtures are not the same as non-chiral material.

BUT (my failures...)

• I haven’t persuaded Costas to model them
• I can’t stop him talking about non-chiral material as “Left-Handed”!

Here’s to another decade (at least!) of trying